ECE 604, Lecture 21

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Contents

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1 Cavity Resonators

1.1 Transmission Line Model

The simplest cavity resonator is formed by using a transmission line. The source end can be terminated by Z_S and the load end can be terminated by Z_L. When Z_S and Z_L are non-dissipative, or reactive, then no energy is dissipitated as a wave is reflected off them. Therefore, if the wave can bounce constructively between the two ends, a coherent solution can exist due to constructive inference, or a resonance solution can exist.

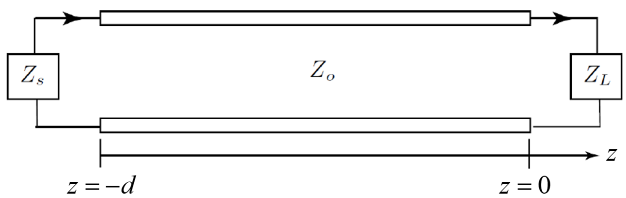

Figure 1:

The transverse resonance condition for 1D problem can be used to derive the resonance condition, namely that

$$1 = \Gamma_S \Gamma_L e^{-2j\beta_z d} \tag{1.1}$$

where Γ_S and Γ_L are the reflection coefficients at the source and the load ends, respectively, β_z the the wave number of the wave traveling in the z direction, and d is the length of the transmission line. For a TEM mode in the transmission line, as in a coax filled with homogeneous medium, then $\beta_z = \beta$, where β is the wavenumber for the homogeneous medium. Otherwise, for a quasi-TEM mode, $\beta_z = \beta_e$ where β_e is some effective wavenumber for a z-propagating wave in a mixed medium. In general,

$$\beta_e = \omega / v_e \tag{1.2}$$

where v_e is the effective phase velocity of the wave in a heterogeneous structure.

When the source and load impedances are replaced by short circuits, or combination of open and short circuits, then the reflection coefficients are -1 for a short, and $+1$ for an open circuit. The above then becomes

$$\pm 1 = e^{-2j\beta_e d} \tag{1.3}$$

When a "+" sign is chosen, the resonance condition is that

$$\beta_e d = p\pi, \quad p = 0, 1, 2, \ldots, \quad \text{or integer} \tag{1.4}$$

For a TEM or a quasi-TEM mode in a transmission line, $p = 0$ is not allowed as the voltage will be uniformly zero on the transmisson line. The lowest mode is when $p = 1$ corresponding to a half wavelength on the transmission line.

Whereas when the line is open at one end, and shorted at the other end in (1.1), the resonance condition corresponds to the "−" sign in (1.3), which gives rise to

$$\beta_e d = p\pi/2, \quad p \quad \text{odd} \tag{1.5}$$

The lowest mode is when $p = 1$ corresponding to a quarter wavelength on the transmission line, which is smaller than that of the short terminated transmission line. Designing a small resonator is a prerogative in modern day electronic design. For example, miniaturization in cell phones calls for smaller components that can be packed into smaller spaces.

A quarter wavelength resonator made with a coax is shown in Figure 2. It is easier to make a short at the left end, but it is hard to make a true open circuit at the right end. A true open circuit means that the current has to be zero. But when a coax is terminated with an open, the electric current does end abruptly. The fringing field at the right end gives rise to stray capacitance through which displacement current can flow in accordance to the generalized Ampere's law. Hence, we have to model the right end termination with a small stray or fringing field capacitance as shown in Figure 2.

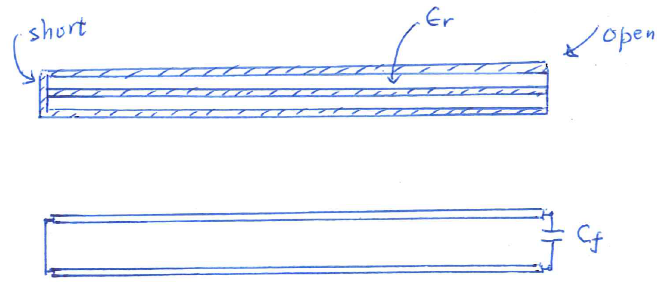

Figure 2:

1.2 Cylindrical Waveguide Resonators

Since a cylindrical waveguide is homomorphic to a transmission line, we can model a mode in this waveguide as a transmission line. Then the termination of the waveguide with either a short or an open circuit at its end makes it into a resonator.

Again, there is no true open circuit in an open ended waveguide, as there will be fringing fields at its open ends. If the aperture is large enough, the open end of the waveguide radiates and may be used as an antenna as shown in Figure 3.

Figure 3: Courtesy of RFcurrent.com.

As previously shown, single-section waveguide resonators can be modeled with a transmission line model using homomorphism with the appropriately chosen β_z. Then, $\beta_z = \sqrt{\beta^2 - \beta_s^2}$ where β_s can be found by first solving a 2D waveguide problem corresponding to the reduced-wave equation.

For a rectangular waveguide, for example,

$$\beta_z = \sqrt{\beta^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \tag{1.6}$$

If the waveguide is terminated with two shorts (which is easy to make) at its ends, then the resonance condition is that

$$\beta_z = p\pi/d, \quad p \text{ integer} \tag{1.7}$$

Together, using (1.6), we have the condition that

$$\beta^2 = \frac{\omega^2}{c^2} = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2 \tag{1.8}$$

The above can only be satisfied by certain select frequencies, and these frequencies are the resonant frequencies of the cavity. The corresponding mode is called the TE_{mnp} mode or the TM_{mnp} mode depending on if these modes are TE to z or TM to z.

The entire electromagnetic fields of the cavity can be found from the scalar potentials previously defined, namely that

$$\mathbf{E} = \nabla \times \hat{z}\Psi_h, \quad \mathbf{H} = \nabla \times \mathbf{E}/(-j\omega\mu\mathbf{H}) \tag{1.9}$$

$$\mathbf{H} = \nabla \times \hat{z}\Psi_e, \quad \mathbf{E} = \nabla \times \mathbf{H}/(j\omega\varepsilon\mathbf{H}) \tag{1.10}$$

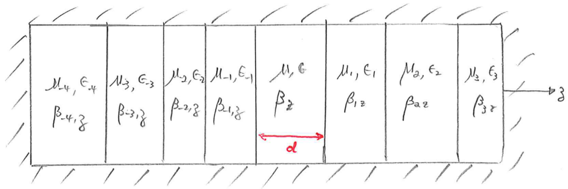

Figure 4:

Since the layered medium problem in a waveguide is the same as the layered medium problem in open space, we can use the generalized transverse resonance condition to find the resonant modes of a waveguide cavity loaded with layered medium as shown in Figure 4. This condition is repeated below as:

$$\tilde{R}_- \tilde{R}_+ e^{-2j\beta_z d} = 1 \tag{1.11}$$

where d is the length of the waveguide section where the above is applied, and $\tilde{R}_-$ and $\tilde{R}_+$ are the generalized reflection coefficient to the left and right of the waveguide section. The above is similar to the resonant condition using the transmission line model in (1.1), except that now, we have replaced the transmission line reflection coefficient with TE or TM generalized reflection coefficients.

Consider now a single section waveguide terminated with metallic shorts at its two ends. Then $R^{TE} = -1$ and $R^{TM} = 1$. Right at cutoff of the cylindrical waveguide, $\beta_z = 0$ implying no z variation in the field. When the two ends of the waveguide is terminated with shorts implying that $R^{TE} = -1$, even though (1.11) is satisfied, the electric field is uniformly zero in the waveguide, so is the magnetic field. Thus this mode is not interesting. But for TM modes in the waveguide, $R^{TM} = 1$, and the magnetic field is not zeroed out in the waveguide, when $\beta_z = 0$.

The lowest TM mode in a rectanglar waveguide is the TM_{11} mode. At the cutoff of this mode, the $\beta_z = 0$ or $p = 0$, implying no variation of the field in the z direction. When the two ends are terminated with metallic shorts, the tangential magnetic field is not shorted out. Even though the tangential electric field is shorted to zero in the entire cavity but the longitudinal electric still exists (see Figures 5 and 6). As such, for the TM mode, $m = 1$, $n = 1$ and $p = 0$ is possible giving a non-zero field in the cavity. This is the TM_{110} mode of the resonant cavity, which is the lowest mode in the cavity if $a > b > d$. The top and side views of the fields of this mode is shown in Figures 5 and 6. The corresponding resonant frequency of this mode satisfies the equation

$$\frac{\omega_{110}^2}{c^2} = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 \tag{1.12}$$

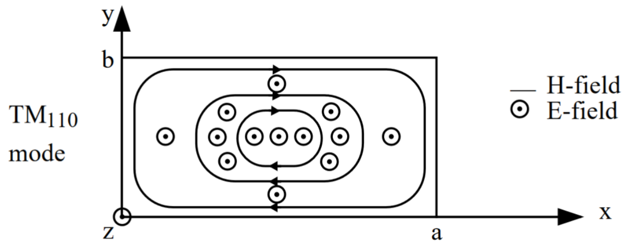

Figure 5:

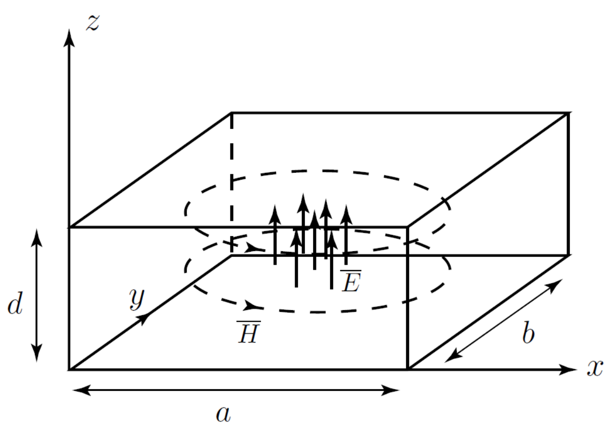

Figure 6: Courtesy of J.A. Kong.

For the TE modes, it is required that $p \neq 0$, otherwise, the field is zero in the cavity. For example, it is possible to have the TE_{101}.

$$\frac{\omega_{101}^2}{c^2} = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2 \tag{1.13}$$

Clearly, this mode has a higher resonant frequency compared to the TM_{110} mode if $d < b$.

The above analysis can be applied to circular and other cylindrical waveguides with β_s determined differently. For instance, for a circular waveguide, β_s is determined differently using Bessel functions, and for a general arbitrarily shaped waveguide, β_s may be determined numerically.

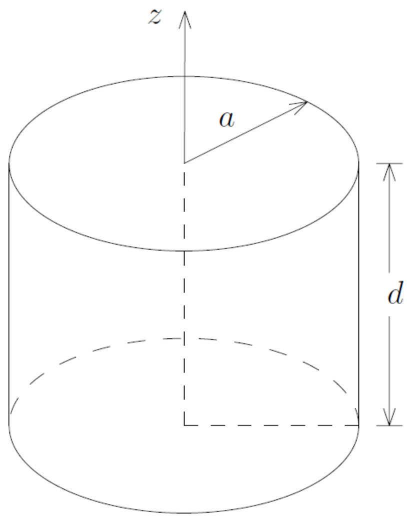

Figure 7:

For s spherical cavity, one would have to analyze the problem in spherical coordinates. The equations will have to be solved by separation of variables using spherical harmonics. Details are given on p. 468 of Kong.

2 Some Applications of Resonators

Resonators in microwaves and optics can be used for designing filters, energy trapping devices, and antennas. As filters, they are used like LC resonators in circuit theory. A concatenation of them can be used to narrow or broaden the bandwidth of a filter. As energy trapping device, a resonator can build up a strong field inside the cavity if it is excited with energy close to its resonance frequency. They can be used in klystrons and magnetrons as microwave sources, a laser cavity for optical sources, or as a wavemeter to measure the frequency of the electromagnetic field. An antenna is a radiator that we will discuss more fully later. The use of a resonator can help in resonance tunneling to enhance the radiation efficiency of an antenna.

2.1 Filters

Microstrip line resonators are often used to make filters. Transmission lines are often used to model microstrip lines in a microwave integrated circuits (MIC). In MIC, due to the etching process, it is a lot easier to make an open circuit rather than a short circuit. But a true open circuit is hard to make as an open ended microstrip line has fringing field at its end as shown in Figure 8. The fringing field gives rise to fringing field capacitance as shown in Figure 2. Then the appropriate Γ_S and Γ_L can be used to model the effect of fringing field capacitance. Figure 9 shows a concatenation of two microstrip resonators to make a microstrip filter. This is like using a concatenation of LC tank circuits to design filters in circuit theory.

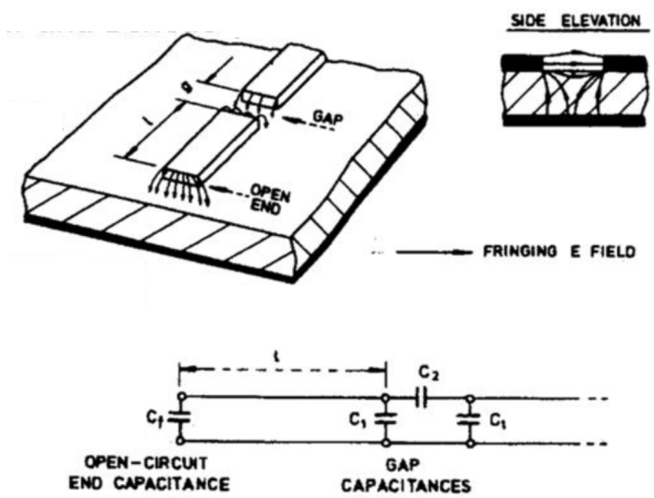

Figure 8: Courtesy of Microwave Journal.

Figure 9: A microstrip filter designed using concatenaed resonators. The connectors to the coax cable are the SMA (sub-miniature type A) connectors (Courtesy of aginas.fe.up.pt).

Optical filters and be made with optical etalon as in a Fabry-Perot resonator, or concatenation of them. This is shown in Figure 10.

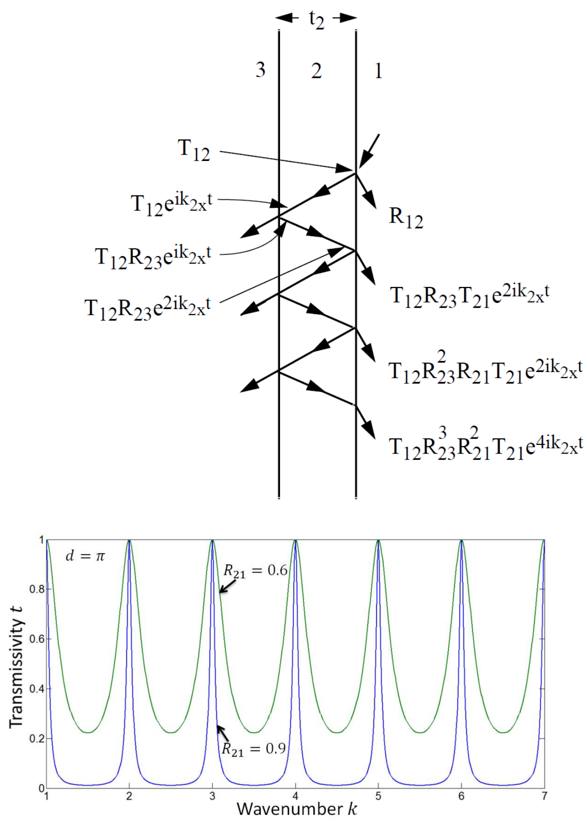

Figure 10:

2.2 Electromagnetic Sources

Microwave sources are often made by transferring kinetic energy from an electron beam to microwave energy. Klystrons, magnetrons, and traveling wave tubes are such devices. However, the cavity resonator in a klystron enhances the interaction of the electrons with the microwave field, causing the field to grow in amplitude as shown in Figure 11.

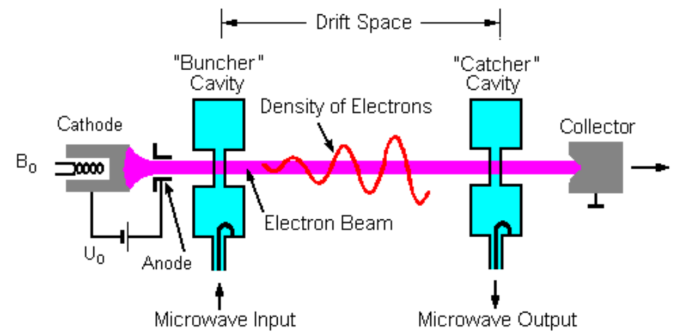

Figure 11: Courtesy of Wiki.

Magnetron cavity works also by transferring the kinetic energy of the electron into the microwave energy. By injecting hot electrons into the magnetron cavity, the cavity resonance is magnified by the kinetic energy from the hot electrons, giving rise to microwave energy.

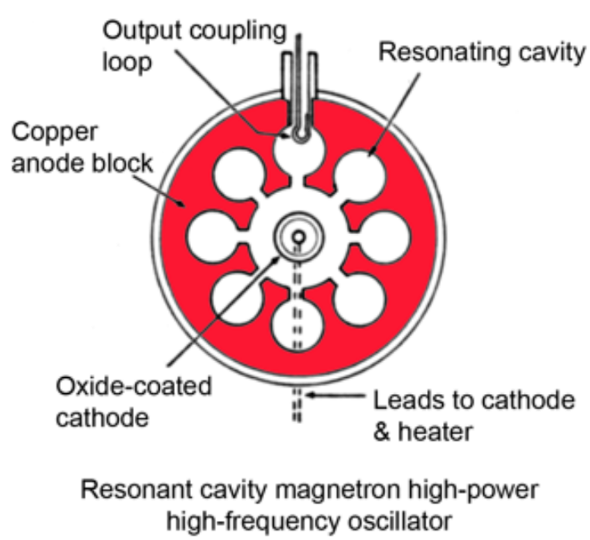

Figure 12: Courtesy of Wiki.

Figure 13 shows laser cavity resonator to enhance of light wave interaction with material media. By using stimulated emission of electronic transition, light energy can be produced.

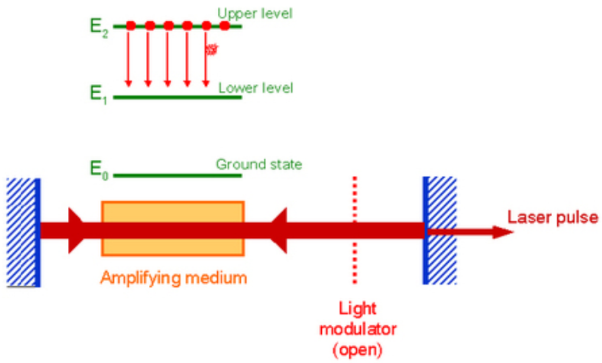

Figure 13: Courtesy of www.optique-ingenieur.org.

Energy trapping of a waveguide or a resonator can be used to enhance the efficiency of a semiconductor laser as shown in Figure 14. The trapping of the light energy by the heterojunctions as well as the index profile allows the light to interact more strongly with the lasing medium or the active medium of the laser. This enables a semiconductor laser to work at room temperature. In 2000, Z I. Alferov and H. Kroemer, together with J.S. Kilby, were awarded the Nobel Prize for information and communication technology. Alferov and Kroemer for the invention of room-temperature semiconductor laser, and Kilby for the invention of electronic integrated circuit (IC) or the chip.

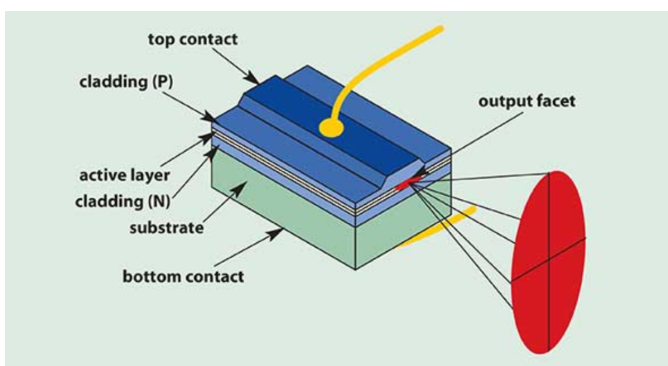

Figure 14:

2.3 Frequency Sensor

Because a cavity resonator can be used as an energy trap, it will siphon off energy from a microwave waveguide when it hits the resonance frequency of the passing wave in the waveguide. This can be used to determine the frequency of the passing wave. Wavemeters are shown in Figure 15 and 16.

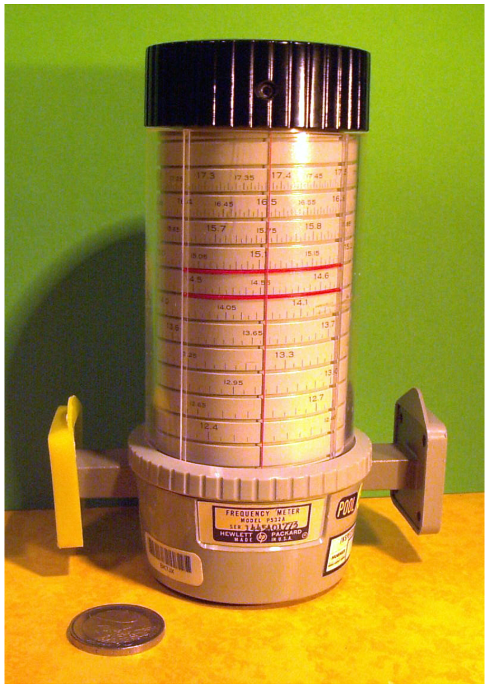

Figure 15: Courtesy of Wiki.

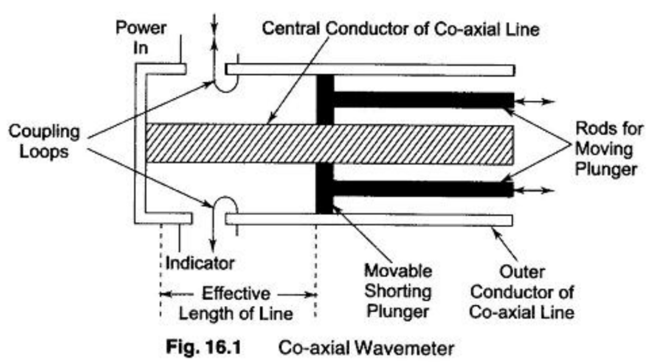

Figure 16: Courtesy of eeeguide.com.